

approx. 2

Entomology Research Branch
P. O. Box 232, Kerrville, Texas

November 23, 1955

To: A. W. Lindquist, Head, Section of Insects Affecting
Man and Animals

From: R. C. Bushland, Entomologist In Charge

Subject: Mathematical Models

I have reviewed your memorandum of November 18 and the proposal submitted by Mr. Langevin.

I am pleased that Dr. Knipling thinks the proposition sufficiently worth while that the Branch will pay half the cost. I have reviewed our budget with Mr. Poeske and he advises me that the Kerrville laboratory can pay the other \$1,400 on the contract if a decision is reached soon enough. We were planning some automobile purchases and if need be, can raise the money by deferring for another year the replacement of a pickup truck. I would just as soon do that since the project is a Kerrville laboratory responsibility and then if later on we needed the money in an emergency funds from your L/A would still be available.

Department of Research Branch
P. O. Box 231, Knoxville, Tenn.

November 23, 1933

A. W. Lindquist, Head, Section of Insect Physiology
Man and Animals

R. C. Marshall, Entomologist in Charge

Subject: Neurological Models

I have reviewed your memorandum of November 12 and the project
submitted by Mr. Langworthy.

I am pleased that Mr. Marshall thinks the proposed work is
worth while and the Bureau will pay half the cost. I have re-
viewed our budget with Mr. West and he advises us that the
Knoxville Laboratory can pay the other \$1,400 on the contract if
a decision is reached soon enough. We were planning some work
on the purchase and it need not be paid the money by deferring
for another year the replacement of a spring frame. I would
just as soon do that since the project is a Knoxville Laboratory
responsibility and when it later on we needed the money in an
emergency funds from your VA would still be available.

UNITED STATES DEPARTMENT OF AGRICULTURE
AGRICULTURAL RESEARCH SERVICE
ENTOMOLOGY RESEARCH BRANCH
BELTSVILLE, MARYLAND
November 18, 1955

NOV 22 1955

To: R. C. Bushland, Box 232, Kerrville, Texas

From: A. W. Lindquist, Head, Insects Affecting Man and
Animals Section

Subject: Mathematical Models

You will recall our conversation with Mr. Langevin in my office during your recent visit. Mr. Langevin has presented a proposal for contract work on this subject. This has been reviewed with Dr. Knipling, and he suggests that if our Section can provide approximately one-half of the \$2800, his office would attempt to supply the other half of the funds. I believe we can take this amount from our LA, but we will not have very much left for assistance to the field stations if an emergency arises.

Please study the enclosed proposal, and if the contract is approved by ARS, we can plan on your working with Mr. Langevin and Dr. Maguire at Orlando the first of next year. I certainly hope we will obtain some worthwhile information from this new approach to entomological problems.

A. W. Lindquist

Enclosure

UNITED STATES DEPARTMENT OF AGRICULTURE
AGRICULTURAL RESEARCH SERVICE
ENTOMOLOGY RESEARCH BRANCH
BELTSVILLE, MARYLAND

November 25, 1955

NOV 28 1955

NOV 25 1955

To: R. C. Bushland, Box 232, Kerrville, Texas

From: A. W. Lindquist, Head, Insects Affecting Man
and Animals Section

Subject: Mathematical Models

Reference is made to your memorandum of November 23 indicating that your L/A can be used for paying \$1400 on the proposed contract with Mr. Langevin on mathematical models dealing with screw-worm research. We hope this will not prevent you from obtaining needed equipment, but it might be easier for you to pay for the contract since transferring funds from this office to the field can be done quickly, whereas transferring funds from the field to this office requires a longer time. We can quickly give you support if you find assistance is needed during the next two or three months.

At the present time, the people working on this contract are proceeding very slowly. We will do everything possible to expedite the signing of the contract so that work can be started.

A. W. Lindquist

MATHEMATICAL MODELLING

OF A

SCREW WORM ERADICATION PROPOSAL

PROPOSAL TOI-W21-55

TO THE

UNITED STATES DEPARTMENT OF AGRICULTURE

TECHNICAL OPERATIONS, INCORPORATED

777 - 14th Street, N.W.

Washington 5, D. C.

UNITED STATES DEPARTMENT OF AGRICULTURE
NATIONAL BUREAU OF ENTOMOLOGY
WASHINGTON, D. C. 20250
February 10, 1955

MATHEMATICAL MODELLING

OF A

SCREW WORM ERADICATION PROPOSAL

PROPOSAL TOI-W21-55

TO THE

UNITED STATES DEPARTMENT OF AGRICULTURE

TECHNICAL OPERATIONS, INCORPORATED

777 - 14th Street, N.W.

Washington 5, D. C.

MATHEMATICAL MODELLING

OF A

SCREW WORM ERADICATION PROPOSAL

INTRODUCTION: A prior proposal, TOI-W18-55, pointed out the curious fact that mathematical models have seldom been employed in entomological studies. This is the more surprising since such models can be expected to provide a theoretical framework around which experimental data can be organized.

As a consequence of our prior proposal discussions with Department of Agriculture personnel have led us to focus our attention for the time being on a specific problem, viz: current proposals and plans for the eradication of the screw worm fly by the release, in selected areas, of numbers of sterile male flies.

The present proposal outlines a mode of approach to the problem which is believed to be at once fruitful and potentially of substantial economic importance.

DISCUSSION: Of the problems surrounding the proposed eradication of the screw worm fly by the release of substantial numbers of sterile male flies none are more important than the influence of the mode of release and the number of sterile flies released upon the time for eradication and indeed upon the very success of the eradication program itself.

Now this is just the kind of situation to which mathematical modelling techniques can be profitably applied. To illustrate the way in which this might be done we have devoted some time to the construction of a preliminary model which describes the proposed screw worm eradication program.

Conceptually this preliminary model is quite simple. To see how it is constructed we may begin by considering the rate of change of the density of eggs/mile². Changes in the egg density can be produced by only three causes:

1. the addition of eggs laid by female flies;
2. the loss of eggs by failure to hatch (death);
3. the loss of eggs by hatching into larva.

It then follows that the rate of change in the egg density is equal to the sum of the rates of change due to these three factors. Similarly the rate change of the larva density must be equal to the sum of the rate of change due to eggs hatching, larvae dying, and larvae pupating.

The same kind of "accounting" can be applied to the rate of change of the pupae density. When the pupal stage is over, however, and the adult

SCREEN WORM ERADICATION PROPOSAL

INTRODUCTION: A prior proposal, T01-W18-25, pointed out the curious fact that mathematical models have seldom been employed in entomological studies. This is the more surprising since such models can be expected to provide a theoretical framework around which experimental data can be organized.

As a consequence of our prior proposal discussions with Department of Agriculture personnel have led us to focus our attention for the time being on a specific problem, viz: current proposals and plans for the eradication of the screw worm fly by the release, in selected areas, of numbers of sterile male flies.

The present proposal outlines a mode of approach to the problem which is believed to be at once fruitful and potentially of substantial economic importance.

DISCUSSION: Of the problems surrounding the proposed eradication of the screw worm fly by the release of substantial numbers of sterile male flies none are more important than the influence of the mode of release and the number of sterile flies released upon the time for eradication and indeed upon the very success of the eradication program itself.

Now this is just the kind of situation to which mathematical modelling techniques can be profitably applied. To illustrate the way in which this might be done we have devoted some time to the construction of a preliminary model which describes the proposed screw worm eradication program.

Conceptually this preliminary model is quite simple. To see how it is constructed we may begin by considering the rate of change of the density of eggs/mile². Changes in the egg density can be produced by only three causes:

1. the addition of eggs laid by female flies;
2. the loss of eggs by failure to hatch (death);
3. the loss of eggs by hatching into larvae.

It then follows that the rate of change in the egg density is equal to the sum of the rates of change due to these three factors. Similarly the rate change of the larvae density must be equal to the sum of the rate of change due to eggs hatching, larvae dying, and larvae pupating.

The same kind of "accounting" can be applied to the rate of change of the pupae density. When the pupal stage is over, however, and the adult

flies emerge the situation becomes more complicated. Roughly, we may expect the emerging flies to be about equally divided between virgin females and virgin males.

To make an accounting of the way the virgin female density changes we have to consider the addition to the virgin female population of the newly emerged female flies, losses due to death, and losses due to mating whereby a female fly acquires the status of a non-virgin female. The first two considerations are straightforward, but the last, losses due to mating, must be looked at in more detail. We have elected to deal with this mating problem by supposing that the rate of change of virgin female density due to this cause is proportional to both the virgin female density and the total (fertile and sterile) male density.

Proceeding in this way it is possible to set up a mathematical model of the physical situation. A mathematical formulation of the model is given, for those interested, in Appendix A to this proposal.

To illustrate how a model of this sort could be used we have set up the relevant equations on analogue computing equipment in our laboratories and examined briefly the consequences of two alternative plans for the release of the sterile male insects. The first release plan consists of the release of sterile males at a steady rate of 90 males/day/mile² for a total of 200 days. The second plan consists of a release rate which begins at 180 males per day/mile² and decreases linearly to zero after 200 days. (It will be readily verified that both plans require precisely the same total number of sterile males for the 200 day period.)

Figure 1 shows graphically the results obtained. Both plots are of egg density versus time; the solid curve representing Plan I release, while the dashed curve shows the density variation for Plan II release.

Figure 2 shows how the results are changed if two of the parameters entering the problem are changed markedly. Specifically, in Figure 2 the adult fly death rate has been increased by nearly 30 per cent while the rate of mating has been increased by 25 per cent.

Even with this rather drastic change in these two constants it is apparent that the Plan II release retains a significant advantage in the rapidity with which it leads to a decrease in the egg density. (This is, of course, a not entirely unexpected result.)

While much more could be said about the uses to which the model could put, the above should, we believe, serve for illustrative purposes. It may, however, be pointed out in closing this discussion that since the costs of breeding, irradiating, and dispensing the sterile male flies are known or calculable, one particularly interesting application of the model would be to use it in conjunction with such costs to devise a minimal cost program.

PROPOSAL: It is proposed that Technical Operations, Incorporated undertake for the Department of Agriculture a study of the application of substantially

flies emerge the situation becomes more complicated. Roughly, we may expect the emerging flies to be about equally divided between virgin females and virgin males.

To make an accounting of the way the virgin female density changes we have to consider the addition to the virgin female population of the newly emerged female flies, losses due to death, and losses due to mating whereby a female fly acquires the status of a non-virgin female. The first two considerations are straightforward, but the last, losses due to mating, must be looked at in more detail. We have elected to deal with this mating problem by supposing that the rate of change of virgin female density due to this cause is proportional to both the virgin female density and the total (fertile and sterile) male density.

Proceeding in this way it is possible to set up a mathematical model of the physical situation. A mathematical formulation of the model is given, for those interested, in Appendix A to this proposal.

To illustrate how a model of this sort could be used we have set up the relevant equations on analogue computing equipment in our laboratories and examined briefly the consequences of two alternative plans for the release of the sterile male insects. The first release plan consists of the release of sterile males at a steady rate of 90 males/day/mile² for a total of 200 days. The second plan consists of a release rate which begins at 180 males per day/mile² and decreases linearly to zero after 200 days. (It will be readily verified that both plans require precisely the same total number of sterile males for the 200 day period.)

Figure 1 shows graphically the results obtained. Both plots are of egg density versus time; the solid curve representing Plan I release, while the dashed curve shows the density variation for Plan II release.

Figure 2 shows how the results are changed if two of the parameters entering the problem are changed markedly. Specifically, in Figure 2 the adult fly death rate has been increased by nearly 30 per cent while the rate of mating has been increased by 25 per cent.

Even with this rather drastic change in these two constants it is apparent that the Plan II release retains a significant advantage in the rapidity with which it leads to a decrease in the egg density. (This is, of course, a not entirely unexpected result.)

While much more could be said about the uses to which the model could put the above should, we believe, serve for illustrative purposes. It may, however, be pointed out in closing this discussion that since the costs of breeding, irradiating, and dispersing the sterile male flies are known or calculable, one particularly interesting application of the model would be to use it in conjunction with such costs to devise a minimal cost program.

PROPOSAL: It is proposed that Technical Operations, Incorporated undertake for the Department of Agriculture a study of the application of substantially

the model proposed in Appendix A to the screw worm eradication problem, with particular reference to overwintering problem in Florida.

DURATION OF STUDY: The program outlined can be completed in approximately two man months.

REPORTS: The contractor will supply a detailed final report at the conclusion of the study.

ESTIMATED COST OF CONTRACT:

Direct labor	\$ 1,000.00
Operating Overhead, @ 75% of Direct Labor	750.00
Direct Charges:	
Travel	400.00
Report Reproduction	<u>75.00</u>
TOTAL PRIME COST	\$ 2,225.00
General Administrative Expense, @ 15% of Prime Cost	<u>333.75</u>
TOTAL COST	\$ 2,558.75
Fixed Fee, @ 10% of Total Cost	<u>255.88</u>
TOTAL CONTRACT PRICE	\$ 2,814.63

TECHNICAL OPERATIONS, INCORPORATED

R. A. Langevin

Proposal No. TOI-W21-55

4 November 1955

the model proposed in Appendix A to the new water sanitation program, with particular reference to overwintering problem in Florida.

DURATION OF STUDY: The program outlined can be completed in approximately two man months.

REPORTS: The contractor will supply a detailed final report at the conclusion of the study.

ESTIMATED COST OF CONTRACT:

Direct Labor	\$ 1,000.00
Operating Overhead, @ 75% of Direct Labor	750.00
Direct Charges:	
Travel	400.00
Report Reproduction	75.00
<u>TOTAL PRIME COST</u>	<u>\$ 2,225.00</u>
General Administrative Expense, @ 15% of Prime Cost	333.75
<u>TOTAL COST</u>	<u>\$ 2,558.75</u>
Fixed Fee, @ 10% of Total Cost	255.88
<u>TOTAL CONTRACT PRICE</u>	<u>\$ 2,814.63</u>

TECHNICAL OPERATIONS, INCORPORATED

R. A. Langevin

Proposal No. TOI-W21-55

4 November 1955

CASE I

ADULT DEATH RATE = .014 PER EIGHT HOURS
MATING PROBABILITY = .0008
SOLID CURVE FOR CONSTANT RELEASE RATE
DASHED CURVE FOR LINEARLY DECREASING
RELEASE RATE

EGG DENSITY IN EGGS PER MILE²

5000

4000

3000

2000

1000

FIGURE I

PLAN I

PLAN II

TIME IN DAYS

0

10

20

30

40

50

60

70

80

90

100

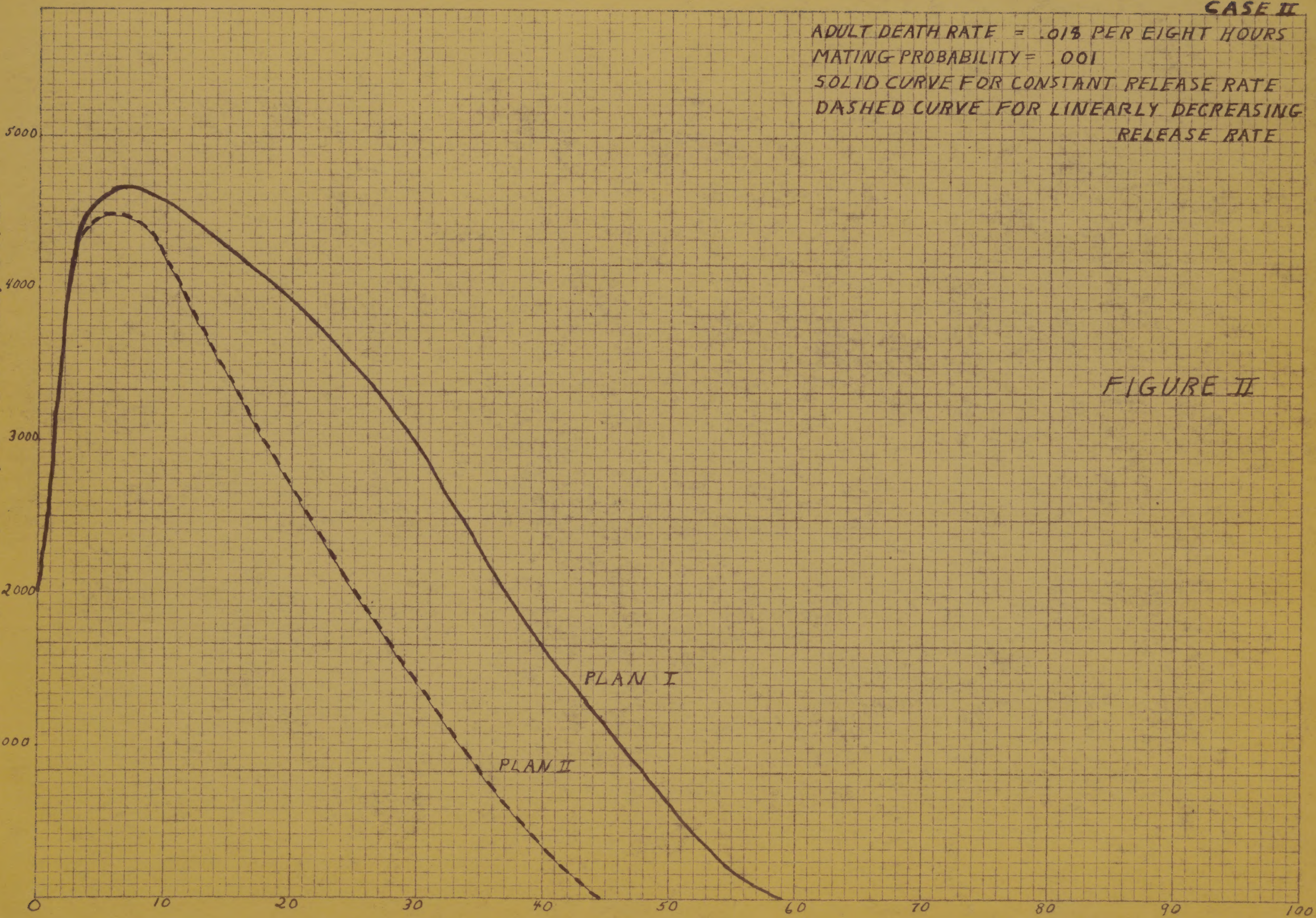
CASE II

ADULT DEATH RATE = .01% PER EIGHT HOURS
MATING PROBABILITY = .001
SOLID CURVE FOR CONSTANT RELEASE RATE
DASHED CURVE FOR LINEARLY DECREASING
RELEASE RATE

EGG DENSITY IN EGGS PER MILE²

FIGURE II

TIME IN DAYS



APPENDIX A

The notation used in the equations below and the constants and boundary conditions used in the illustrative problem discussed in the text of the proposal are tabulated below in Table 1.

$P_1(t)$ = virgin female density, individuals/mile ²	$P_1(0) =$	11
$P_2(t)$ = fertile female density, individuals/mile ²	$P_2(0) =$	89
$P_3(t)$ = sterile male density, individuals/mile ²	$P_3(0) =$	see below
$P_4(t)$ = fertile male density, individuals/mile ²	$P_4(0) =$	100
$P_5(t)$ = fertile egg density, individuals/mile ²	$P_5(0) =$	2,000
$P_6(t)$ = larval density, individuals/mile ²	$P_6(0) =$	21,000
$P_7(t)$ = pupal density, individuals/mile ²	$P_7(0) =$	35,000
f	fraction of female eggs	= .5
β	eggs laid/8 hours/female	= 25
d_A	fraction of adult population dying in 8 hours	= .014
d_E	fraction of egg population dying in 8 hours	= .1
d_L	fraction of larva population dying in 8 hours	= .045
P	probability a female mates in 8 hours for unit male density	= .001
P_8	fraction of eggs hatching in 8 hours	= .9
P_9	fraction of larvae pupating in 8 hours	= .05
P_{10}	fraction of pupae emerging in 8 hours	= .03

Then the relevant equations accounting for the rate of change of the density of each kind of individual are as follows:

$$\frac{d\rho_1}{dt} = f P_7 \rho_7 - d_H \rho_1 - P_1 \rho_1 (\rho_3 + \rho_4)$$

$$\frac{d\rho_2}{dt} = P_1 \rho_1 \rho_4 - d_H \rho_2$$

$$\frac{d\rho_3}{dt} = Q - d_H \rho_3$$

$$\frac{d\rho_4}{dt} = (1-f) P_7 \rho_7 - d_H \rho_4$$

$$\frac{d\rho_5}{dt} = P_2 \rho_2 - d_E \rho_5 - P_5 \rho_5$$

$$\frac{d\rho_6}{dt} = P_5 \rho_5 - d_L \rho_6 - P_6 \rho_6$$

$$\frac{d\rho_7}{dt} = P_6 \rho_6 - P_7 \rho_7$$

Where Q is the rate of release of sterile male insets.

